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SGMCK

(An Autonomous College)

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Rayat Shikshan Sanstha's

Sadguru Gadge Maharaj College, Karad

(An Autonomous College)

029799

SGMCK

(AN EMPOWERED AUTONOMOUS COLLEGE)

Information to be filled by Student

(विद्यार्थ्याने भरावयाचा रकाना)

Day and Date : Saturday, 01/11/2025

Language of Answer : English

Examination : Bsc III Sem V 2025-26

Question Paper Code No. : MJ-BST23-503

Subject : Statistics paper XI

Paper No. V

Section : -

(Office Use)

Q. No.	Examiner Marks	Moderator Marks
1	08	
2	16	
3	16	
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Total	40	
Signature		

Q.C. - O. K.

IMPORTANT

As per Maharashtra Act No. XXXI of 1982 (7 & 8) whoever is found in or near an examination hall by the invigilator or any other person appointed to supervise the conduct of the examination, copying answers to the question paper set at the examination, from any book, notes or answer papers of other candidates, of appearing at the examination for any other candidates or using any other unfair means, shall, on conviction, be punished with imprisonment for a term which may extend to six months, or with fine which may extend to five hundred rupees or with both.

Whoever abets any offence punishable under this Act shall be punishable with the punishment provided for the offence.

सूचना : विद्यापीठ नियमानुसार जो कोणी परिक्षा हॉलजवळ किंवा हॉलमध्ये पर्यवेक्षकांना किंवा परीक्षेसाठी नेमलेल्या कर्मचाऱ्यांना उत्तरांची पुस्तक नोट्स किंवा अन्य विद्यार्थ्यांच्या उत्तरपत्रिकेतून नक्कल करताना आढळून येईल किंवा जवळ परीक्षा गैरव्यवहारासाठी वापरता येईल असे अक्षेपार्ह साहित्य बाळगत असेल तर सदरची बाब परीक्षा प्रमाद मानून संबंधित व्यक्ति शिक्षेस पात्र राहील.

महत्वाच्या सूचना

१. उत्तरपत्रिकेमध्ये अन्यत्र कोठेही बैठक क्रमांक (दिलेल्या जागेखेरीज) लिहू नये, तसेच अन्यत्र कोणत्याही प्रकारच्या खुणा करू नये अशा खुणा आढळल्यास तो ओळख पटविण्याचा प्रयत्न मानन्यात येईल व असा परीक्षा प्रमाद शिक्षेस पात्र राहील.
२. उत्तरास प्रश्न क्रमांक व उपप्रश्न क्रमांक लिहणे अनिवार्य आहे.
३. पूर्ण प्रश्नांचे उत्तर सलग व सुवाच्च अक्षरात लिहावे.
४. अवाचनीय अक्षरातील उत्तर शुन्य गुणासाठी ग्राह्य मानण्यात येईल.
५. आकृत्याखेरीज सर्व उत्तरासाठी फक्त निळ्या शाईचा वापर करावा.

IMPORTANT

1. Do not write seat number anywhere except at the space provided for it. If seat number or any marks, symb which indicates identity is/are found anywhere, it will be treated as malpractice and lapse. Such malpractice lapse will be punishable under Maharashtra University Act 1994.
2. It is necessary to write the Question No. and sub Question Number before answer.
3. Write the complete answer in sequential manner and then start the next question in readable handwriting.
4. Answer in an illegible & undecipherable handwriting are liable to be marked as zero.
5. Use of blue color ink (expect diagrams) is permitted.

Q. No.	Verificaton Marks	Revaluation Ma
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Q. No.

Q.1.

i)
→ a) Cluster Sampling

ii)
→ b) n/N

iii)
→ c) More reliable than simple random sampling.

iv)
→ b) When sampling Frame is not available.

v)
→ c) Both (a) and (b)

vi)
→ c) A list of sampling units in the population.

vii)
→ c) Both (a) & (b)

viii)
→ d) None of these

Q. No.	2)	a	b	TOTAL				TOTAL
Marks		08	08	16				

Q. No.

Q.2.

a)

→ Stratified random sampling :- For estimating population ~~wt~~ mean with more precision we have to reduce variance ~~V~~. One way is to increase the sample size and other way is to devide the population into several groups such that each of them is more homogeneous than entire population then select a random sample from each of this group. This procedure is called stratified random sampling.

First devide the population into groups such that -

1) Sample units within each group are as homogeneous as possible.

2) Sample units between each grous are as heterogeneous as possible.

Each of the group is called stratum & the groups are called strata.

Statemen :- Stratified random sample mean is an unbiased estimator of population mean.

i.e. $E(\bar{y}_{st}) = \bar{Y}$

Proof :- We know that

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Q. No.

$$\bar{y}_{st} = \sum_{i=1}^{K} P_i \bar{y}_i$$

$$E(\bar{y}_{st}) = \sum_{i=1}^{K} P_i \; E(\bar{y}_i)$$

We know that sample is draw from each stratum by SRSWOR method

$$E(\bar{y}_i) = \bar{Y}_i$$

$$E(\bar{y}_{st}) = \sum_{i=1}^{K} P_i \; E(\bar{y}_i)$$

Where $\bar{Y}_i$ is the i^{th} stratum mean from the population.

$$\therefore \; E(\bar{y}_{st}) = \sum_{i=1}^{K} P_i \; E(\bar{y}_i)$$

$$= \sum_{i=1}^{K} \frac{N_i}{N} \bar{Y}_i$$

$$= \frac{1}{N} \sum_{i=1}^{K} N_i \bar{Y}_i \quad \cdots (Y = N\bar{Y})$$

$$= \frac{1}{N} \left[N_1 \bar{Y}_1 + N_2 \bar{Y}_2 + \cdots + N_K \bar{Y}_K \right]$$

$$= \frac{1}{N} \; Y$$

Q. No.

$$E(\bar{y}_{st}) = \frac{M}{N} \sum_{i=1}^{K} \sum_{j=1}^{N_i} y_{ij}$$

$$E(\bar{y}_{st}) = \bar{Y}$$

∴ Which shows that stratified random sample mean is an unbiased estimator of population mean.

Variance of estimator of population mean under proportional allocation is

$$V(\bar{y}_{st}) = \sum_{i=1}^{K} \left(\frac{1}{n_i} - \frac{1}{N_i}\right) P_i^2 S_i^2$$

$$= \sum_{i=1}^{K} \frac{P_i^2 S_i^2}{n_i} - \sum_{i=1}^{K} \frac{P_i^2 S_i^2}{N_i}$$

$$= \sum_{i=1}^{K} \frac{P_i^2 S_i^2}{n_i} - \sum_{i=1}^{K} \frac{\left(\frac{N_i}{N}\right)^2 S_i^2}{N_i}$$

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$$= \sum_{i=1}^{K} \frac{P_i^2 S_i^2}{n_i} - \sum_{i=1}^{K} \frac{\left(\frac{N_i}{N}\right)^2 S_i^2}{N_i}$$

put $n_i = nP_i$

$$= \sum_{i=1}^{K} \frac{P_i^2 S_i^2}{nP_i} - \sum_{i=1}^{K} \frac{P_i S_i^2}{N}$$

$$= \sum_{i=1}^{K} \frac{P_i S_i^2}{n} - \sum_{i=1}^{K} \frac{P_i S_i^2}{N}$$

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$$\cancel{E(\bar{y}} \; V(\bar{y}_{st}) = \sum_{i=1}^{k}\left(\frac{1}{n}-\frac{1}{N}\right)P_i S_i^2$$

$$\therefore \; S.E.(\bar{y}_{st}) = \sqrt{V(\bar{y}_{st})}$$

$$\therefore \; \boxed{S.E.(\bar{y}_{st}) = \sqrt{\sum_{i=1}^{k}\left(\frac{1}{n}-\frac{1}{N}\right)P_i S_i^2}}$$

b)

→ Random Sampling :- SRS is the method of Sampling in which sample from population is obtained unit by unit in such a way that every unit of the population has an equal & independent chance of selecting in each draw is called simple random sampli-ng or Rand sampling or equal probability sampling.

Statement :- Sample mean square (s^2) is an unbiased estimator of population mean square (S^2). i.e. $E(s^2) = S^2$

Proof :- We know that

$$s^2 = \frac{1}{n-1}\left(\sum_{r=1}^{n}(y_r-\bar{y})^2\right)$$

$$= \frac{1}{n-1}\sum_{r=1}^{n} y_r - \bar{y}^2$$

Q. No.				TOTAL				TOTAL
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Q. No.

$$s^2 = \frac{1}{n-1} \sum_{r=1}^{n} y_r^2 - n\bar{y}^2$$

$$E(s^2) = E\left[\frac{1}{n-1} \sum_{r=1}^{n} y_r^2 - n\bar{y}^2\right]$$

$$= \frac{1}{n-1} E\left(\sum_{r=1}^{n} y_r^2\right) - nE(\bar{y}^2) \quad \text{---} (1)$$

We know that

$$V(\bar{y}) = E(\bar{y}^2) - [E(\bar{y})]^2$$

$$\frac{N-n}{Nn} S^2 = E(\bar{y}^2) - \bar{Y}^2$$

$$E(\bar{y}^2) = \frac{N-n}{Nn} S^2 + \bar{Y}^2 \quad \text{---} (2)$$

$$E\left(\sum_{r=1}^{n} y_r^2\right) = E\left(\sum_{i=1}^{N} y_i^2 a_i\right)$$

$$= \sum_{i=1}^{N} y_i^2 E(a_i)$$

$$= \sum_{i=1}^{N} y_i^2 \left(\frac{n}{N}\right)$$

$$E\left(\sum_{r=1}^{n} y_r^2\right) = \frac{n}{N} \sum_{i=1}^{N} y_i^2 \quad \text{---} (3)$$

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from eq^n ② & eq^n ③ eq^n ① becomes

$$E(s^2) = \frac{1}{n-1}\left[\frac{n}{N}\sum_{i=1}^{n} y_i^2 - n\left(\frac{N-n}{Nn}\right)S^2 + \bar{Y}^2\right]$$

$$= \frac{1}{n-1}\left[\frac{n}{N}\sum_{i=1}^{n} y_i^2 - \left(\frac{N-n}{N}\right)S^2 - n\bar{Y}^2\right]$$

$$= \frac{1}{n-1}\left[\frac{n}{N}\sum_{i=1}^{n} y_i^2 - n\bar{Y}^2 - \frac{N-n}{N}S^2\right]$$

$$= \frac{1}{n-1}\left[\frac{n}{N}\left(\sum_{i=1}^{N} y_i^2 - N\bar{Y}^2\right) - \frac{N-n}{N}S^2\right]$$

$$= \frac{1}{n-1}\left[\frac{n}{N}\left(\sum_{i=1}^{N} y_i^2 - N\bar{Y}^2\right) - \frac{N-n}{N}S^2\right]$$

We have

$$S^2 = \frac{1}{N-1}\left(\sum_{i=1}^{N} y_i^2 - N\bar{Y}^2\right)$$

$$(N-1)S^2 = \sum_{i=1}^{N} y_i^2 - N\bar{Y}^2$$

$$E(s^2) = \frac{1}{n-1}\left[\frac{n}{N}(N-1)S^2 - \frac{N-n}{N}S^2\right]$$

$$= \frac{S^2}{(n-1)N}\left[n(N-1) - (N-n)\right]$$

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Q. No.

$$E(\$^2) = \frac{S^2}{(n-1)N}\left[nN - n - N + n\right]$$

$$= \frac{S^2}{(n-1)N}\left[nN - N\right]$$

$$= \frac{S^2}{(n-1)N}\left[N(n-1)\right]$$

$$E(\$^2) = S^2$$

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∴ Which shows that sample mean squar ($\$^2$) is an unbiased estimator of population mean square (S^2)

Q. No.	3	a	c	TOTAL	d	e		TOTAL
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Q. No.

Q.3.

a)

→ For SRSWR :-

statement :- sample proportion is an un-biased estimator of population proportion.

i.e. $E(P) = P$

Proof :- We know that in SRSWR sample mean is an unbiased estimator of population mean.

i.e. $E(\bar{y}) = \bar{Y}$

$$E\left[\frac{1}{n}\sum_{r=1}^{n} y_r\right] = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$E\left[\frac{1}{n}(a)\right] = \frac{1}{N}(A)$$

$$\therefore E\left(\frac{a}{n}\right) = \frac{A}{N}$$

$$E(P) = P$$

∴ Which show that sample proportion is an unbiased estimator of population proportion.

For SRSWOR :-

Q. No.			TOTAL				TOTAL
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statement :- Sample proportion is an unbiased estimator of population proportion.

i.e. $E(P) = P$

Proof :- We know that in SRSWOR sample mean is an unbiased estimator of population mean.

i.e. $E(\bar{y}) = \bar{Y}$

$$E\left[\frac{1}{n}\sum_{r=1}^{n} y_r\right] = \frac{1}{N}\sum_{i=1}^{N} Y_i$$

$$E\left[\frac{1}{n}(a)\right] = \frac{1}{N}(A)$$

$$E\left(\frac{a}{n}\right) = \frac{A}{N}$$

$$E(P) = P$$

Which shows that sample proportion is an unbiased estimator of population proportion

c) → Approximate variances of regression and ratio methods are

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$$V(\bar{y})_{Reg} = V(\hat{\bar{y}})_{reg} = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 (1-\rho^2)$$

$$V(\bar{y})_R = V(\hat{\bar{y}})_R = \left(\frac{1}{n} - \frac{1}{N}\right)(S_y^2 + R^2 S_x^2 - 2R S_{xy})$$

$$= \left(\frac{1-F}{n}\right)(S_y^2 + R^2 S_x^2 - 2R S_{xy})$$

We have

$$\rho = \frac{S_{xy}}{S_x S_y}$$

$$\rho S_x S_y = S_{xy}$$

$$V(\bar{y})_R = V(\hat{\bar{y}})_R = \left(\frac{1}{n} - \frac{1}{N}\right)(S_y^2 + R^2 S_x^2 - 2R S_{xy})$$

$$= \left(\frac{1-F}{n}\right)(S_y^2 + R^2 S_x^2 - 2R \rho S_x S_y)$$

If variance of regression estimator is better ~~If~~ ~~variance~~ than ratio estimator.

If $V(\bar{y})_{Reg} \le V(\bar{y})_R$

$$\left(\frac{1-F}{n}\right) S_y^2 (1-\rho^2) \le \left(\frac{1-F}{n}\right)(S_y^2 + R^2 S_x^2 - 2R S_{xy})$$

$$\left(\frac{1-F}{n}\right) S_y^2 - \left(\frac{1-F}{n}\right) S_y^2 \rho^2 \le \left(\frac{1-F}{n}\right) S_y^2 + \left(\frac{1-F}{n}\right) R^2 S_x^2 - \left(\frac{1-F}{n}\right) 2R S_{xy}$$

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$$-\left(\frac{1-F}{n}\right) S_y^2 \rho^2 \leq \left(\frac{1-F}{n}\right) R^2 S_x^2 - \left(\frac{1-F}{n}\right) 2RS_{xy}$$

$$-S_y^2 \rho^2 \leq R^2 S_x^2 - 2RS_{xy}$$

$$R^2 S_x^2 - 2RS_{xy} + S_y^2 \rho^2 \geq 0$$

$$R^2 S_x^2 - 2R\rho S_x S_y + S_y^2 \rho^2 \geq 0$$

04 $$(RS_x - \rho S_y)^2 \geq 0$$

Which always holds. The regression estim-ate is always superior to the ratio esti-mate.

d)

→ Situations where systematic sampling is use

1) Listing of population members in logical order (e.g. alphabetical, chronological)

2) Equal spacing between items (e.g. every n^{th} item).

3) Data collection from continous distribution

4) Quality control inspections.

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5) large population where random sampling is impracticable.

6) Reasearch studies with structured data collection.

7) Auditing & accounting ~~-~~ selecting every n^{th} transiction or account for view.

8) Manufacturing :- selecting every n^{th} product on an assembly line.

9) Medical research :- Selecting patients at regular interval from hospital database.

10) Customer Survey :- Contacting every n^{th} customer at regular interval from mailing list.

11) Sampling posts on social media analysis or comments from regular interval.

e) Advantages of Sampling method over census method :-

1) Reduced cost :- If data is collected pre only small fraction of the population expenditure are smaller than complete census is attempted with large population. We can get

more accurate results from small fraction of the population.

2) Greater speed / less time :- Data from small fraction can collected with maximum speed than complete enumration. When information is urgently required we consider only sampling method.

3) Greater accuracy :- For sampling survey we can employ persons of high quality trained they can make carefull supevision of field work. Sample may produce more accurate results than complete census or complete enumration.

4) Practicable :- In some situations like blood testing or testing the life of ~~eq~~ some equipments ~~we~~ only sampling is way to collect the data & census method is feasible.

5) Greater scope :- In certain types, an enquiry we have to use highly trained persons or some speedy equipments in this case census method is impracticable thus survey that as sampling have more scope.